

螺旋锥齿轮边缘接触分析*

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【摘要】 边缘接触是一种轮齿齿顶边缘传递运动的现象。这时,接触区在齿根或齿顶处产生一条硬印。常规的齿面接触分析(TCA)结果是一组间断的误差曲线和不完整的接触区,未能反映一个完整的接触过程。本文提出了描述边缘接触这一现象的具体方法。可得到完整的传动误差曲线和接触区图,为进一步的边缘接触振动分析打下了基础

主题词: 螺旋锥齿轮 边缘 接触 传动 误差

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1 引言

众所周知,齿轮副靠齿面啮合传递运动,但有时也会发生轮齿齿顶边缘传递运动的现象,我们称这一现象为边缘接触。对于弧齿锥齿轮和准双曲面齿轮,边缘接触是极易发生的。机床调整参数的不合理、安装误差及轮齿变形等因素都可能导致边缘接触的发生。当发生边缘接触时,齿轮副的运动是不稳定的,将产生很大的噪声和振动。许多文献都对发生边缘接触这一现象的可能性进行了论述,但都没能给出具体计算方法和计算结果。对于会发生边缘接触的齿轮副,常规的齿面接触分析(TCA)得到的传动误差曲线是间断的,但这并不意味着齿轮副传递运动是间断的。实际上,这时由一个齿轮的齿顶边缘和另一齿轮的齿面啮合继续传递运动。本文提出了描述边缘接触这一现象的具体方法,给出了完整的传动误差曲线和接触区图。本文所提出的方法虽然主要是针对螺旋锥齿轮的,但其基本概念同样可适用于直齿轮和斜齿轮。本文最后给出了上述方法在弧锥齿轮上应用的数值示例。

2 齿面接触分析

建立如图 1 所示的坐标系,坐标系 S_1 和 S_2 分别与小轮和大轮固定在一起,其坐标原点 O_1 和 O_2 分别为小轮和大轮的节锥顶点。参数 φ_1 和 φ_2 分别为小轮和大轮转角。坐标系 S_h 和 S_d 分别固定在箱体上,其原点 O_h 和 O_d 分别与 O_1 和 O_2 重合。

设在 S_1 和 S_2 坐标系中,大轮和小轮的齿面方程和单位法向量分别为:

$$\vec{r}_i = \vec{r}_i(u_i, \theta) \quad (i = 1, 2) \quad (1)$$

$$\vec{n}_i = \vec{n}_i(u_i, \theta) \quad (i = 1, 2) \quad (2)$$

其中: u_i 和 θ 为齿面参数。 S_h 坐标系中,小轮和大轮的齿面方程及单位法向量为:

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$$\begin{cases} \vec{r}_h^{(i)} = [M_{hi}] = \vec{r}_h^{(i)}(u_i, \theta, \varphi) \\ \vec{n}_h^{(i)} = [L_{hi}] = \vec{n}_h^{(i)}(u_i, \theta, \varphi) \end{cases} \quad (i = 1, 2) \quad (3)$$

其中: $[M_{hi}]$ 和 $[L_{hi}]$ 是坐标系 S_i 和 S_h 的变换矩阵。

一对齿轮啮合时, 接触点有共同的位置矢量和法矢量:

$$\begin{cases} \vec{r}_h^{(1)}(u_1, \theta_1, \varphi) = \vec{r}_h^{(2)}(u_2, \theta_2, \varphi) \\ \vec{n}_h^{(1)}(u_1, \theta_1, \varphi) = \vec{n}_h^{(2)}(u_2, \theta_2, \varphi) \end{cases} \quad (i = 1, 2) \quad (4)$$

式(4)便为 TCA 分析的基本方程。由于 $|\vec{n}_h^{(1)}| = |\vec{n}_h^{(2)}| = 1$, 所以式(4)仅有五个独立标量方程, 选择 φ 作为输入参数, 可用求解非线性方程组的子程序求解出其它 5 个参数: $u_1(\varphi)$, $\theta_1(\varphi)$, $u_2(\varphi)$, $\theta_2(\varphi)$, $\varphi(\varphi)$, 按步长给定 φ , 从而可求出啮合过程中的接触路径, 进而可确定接触椭圆。

使用函数 $\varphi(\varphi)$, 可以确定传动误差函数:

$$\Delta\varphi(\varphi) = \varphi_2(\varphi) - (N_1/N_2)\varphi \quad (5)$$

其中: N_1 和 N_2 为小轮和大轮齿数。

对于边缘接触情况, 为方便起见, 我们以小轮为例, 即小轮齿顶边缘与轮齿面发生接触。这时, 小轮齿顶边缘和大轮齿面在接触点有共同的位置矢量, 并且在接触点小轮齿顶边缘的切向量必然垂直于大轮齿面的法向量, 即:

$$\begin{cases} \vec{r}_h^{(1)}(u_1(\theta_1), \theta_1, \varphi) = \vec{r}_h^{(2)}(u_2, \theta_2, \varphi) \\ (\partial \vec{r}_h^{(1)} / \partial \theta_1) \cdot \vec{n}_h^{(2)}(u_2, \theta_2, \varphi) = 0 \end{cases} \quad (6)$$

这里 $\vec{r}_h^{(1)}(u_1(\theta_1), \theta_1, \varphi)$ 为在 S_h 坐标系中小轮齿顶边缘在接触点的位置矢量, 它是小轮齿面与小轮顶锥的交线。 $\partial \vec{r}_h^{(1)} / \partial \theta_1$ 为小轮齿顶边缘的切向量。

方程组(6)有 4 个独立的非线性方程, 当把 φ 作为输入参数时, 理论上可以求解出 4 个未知量: $\theta_1(\varphi)$, $\theta_2(\varphi)$, $u_2(\varphi)$, $\varphi(\varphi)$, 这里函数 $u_1 = u_1(\varphi)$ 在轮齿边缘是已知的。

方程组(6)式几乎被看作是求解边缘接触的标准方程组^[1~3], 但作者至今还没看到此方程求解的具体方法和具体结果。分析其原因可能是由于偏导数 $\partial \vec{r}_h^{(1)} / \partial \theta_1$ 的具体表示是十分困难的。本文用齿面法矢量与顶锥法矢量的叉积来表示边缘曲线的切向量, 使得计算大为简化。

3 边缘接触分析

3.1 小轮齿顶边缘与大轮齿面接触

在 S_1 坐标系中, 小轮顶锥与小轮齿面相交产生的边缘曲线应满足(7)式(图 2):

$$\tan \delta_{a1} = \frac{\sqrt{Y_1^2 + Z_1^2}}{X_1 - D_{a1}} \quad (7)$$

这里 δ_{a1} 为小轮顶锥角, D_{a1} 为节锥顶点与顶锥顶点的距离。

由图 2 可知, 顶锥的法向量为:

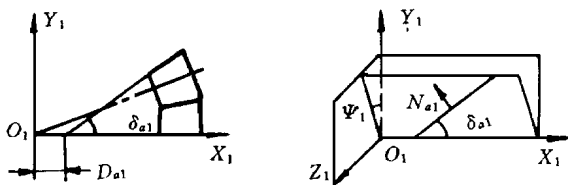


图 2 小轮坐标系

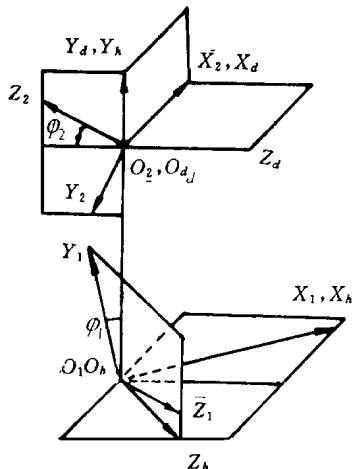


图 1 大、小轮相对位置坐标系

$$\vec{N}_{a1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_1 & \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{bmatrix} \begin{Bmatrix} -\sin\delta_{a1} \\ \cos\delta_{a1} \\ 0 \end{Bmatrix} \quad (8)$$

其中 $\psi_1 = \tan^{-1}(Z_1/Y_1)$ (9)

在 S_1 坐标系中, 设小轮齿面的法向量为 $\vec{N}_1$, 则小轮顶锥与小轮齿面相交产生的边缘曲线切向量为:

$$\vec{T}_1 = \vec{N}_{a1} \times \vec{N}_1 \quad (10)$$

将 $\vec{T}_1$ 由 S_1 坐标系变换到 S_h 坐标系中(图 1), 得:

$$\vec{T}_h^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_1 & \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{bmatrix} \vec{T}_1 \quad (11)$$

因此可得边缘接触求解方程组:

$$\vec{r}_h^{(1)} - \vec{r}_h^{(2)} = 0, \quad \vec{T}_h^{(1)} \times \vec{N}_h^{(2)} = 0, \quad (X_1 - D_{a1}) \tan\delta_{a1} - \sqrt{Y_1^2 + Z_1^2} = 0 \quad (12)$$

其中第一式表示两齿面在接触点位置矢量相同, 第二式表示小轮齿顶边缘曲线切向量与大轮齿面法线方向垂直, 第三式为式(7)的改写形式。(12)式五个非线性标量方程, 当把 φ 作为输入参数时, 可求出五个未知量: $\theta(\varphi)$, $\theta_2(\varphi)$, $u_2(\varphi)$, $\varphi_2(\varphi)$ 及 $\psi(\varphi)$ [或 $u_1(\varphi)$]。

3.2 大轮齿顶边缘与小轮齿面接触

大轮齿顶边缘与小轮齿面接触的计算公式和小轮齿顶边缘与大轮齿面接触的计算公式基本相同, 只要将上小节公式中的上、下 1 改为 2, 2 改为 1 即可, 但大轮顶锥与大轮齿面交线的切向量 $\vec{T}_2$ 由 S_2 坐标系变换到 S_h 坐标系的变换矩阵是不同的, 由图 1 可知, 变换公式为:

$$\vec{T}_h^{(2)} = \begin{bmatrix} \cos\Gamma & 0 & \sin\Gamma \\ 0 & 1 & 0 \\ -\sin\Gamma & 0 & \cos\Gamma \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\varphi_2 & \sin\varphi_2 \\ 0 & -\sin\varphi_2 & -\cos\varphi_2 \end{bmatrix} \vec{T}_2 \quad (13)$$

3.3 求解过程

经过以上的准备, 我们可以得到一个完整的 TCA 求解过程如下: (1) 当大轮和小轮在齿面上啮合时, 可由方程组(4)求出两齿面啮合过程中接触各点的 5 个参数: $u_1(\varphi)$, $\theta(\varphi)$, $u_2(\varphi)$, $\theta_2(\varphi)$, $\varphi_2(\varphi)$ 。进而可求得传动误差及确定接触椭圆^[1,4,5]。(2) 在啮合过程中, 如果啮合点超出齿顶边缘, 则改用方程组(12)继续求解(对于大轮齿顶边缘与小轮齿面接触见 3.2 节)。

4 数值示例

表 1 所列为例的轮坯参数。

表 1 轮坯参数

	齿数	压力角	螺旋方向	顶隙	全齿高	齿顶高	齿根高	分锥角	根锥角	顶锥角
小轮	11	20°	左	0.81	8.11	5.24	2.87	15°1'	13°20'	19°21'
大轮	14	20°	右	0.81	8.11	2.87	5.24	74°59'	70°39'	76°40'

轴交角为 90° , 外锥距为 90.07, 齿宽为 27.03。齿轮的加工参数略。

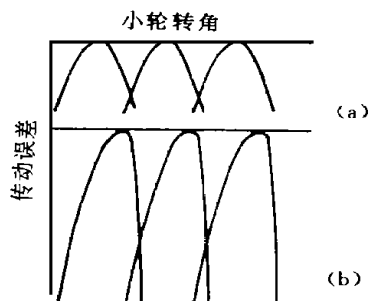


图 3 传动误差曲线(a) $V=0$ (b) $V=0.12$

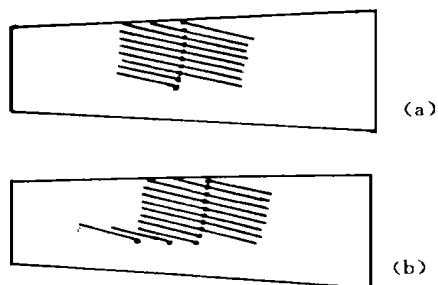


图 4 接触区图(a) $V=0$ (b) $V=0.12$

图 3(a)和(b)分别为垂直位移 $V=0$ 和 $V=0.12$ 时的传动误差曲线,图 4(a)和(b)分别为垂直位移 $V=0$ 和 $V=0.12$ 的接触区图。从图中可以看出, $V=0$ 中未发生边缘接触,而 $V=0.12$ 中发生了边缘接触,这里我们得到了完整的传动误差曲线和接触区图。

5 结束语

本文成功地解决了弧齿锥齿轮和准双曲面齿轮边缘接触分析问题,得到了完整的传动误差曲线和接触区图。本文最后给出了一个弧齿锥齿轮的计算示例。本文所提出的方法虽然主要是针对点接触齿轮副的边缘接触,但其基本概念同样适用于线接触的齿轮副,如直齿轮和斜齿轮。边缘接触分析这一问题的解决,为进一步的边缘接触振动分析打下了基础。

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A STUDY ON AVERAGED HEAT TRANSFER CHARACTERISTICS OF A ROTATING CAVITY WITH AXIAL FRINGE INFLOW

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ABSTRACT A turbine cooling configuration of an aero-engine was simplified to a rotating cavity with an axial fringe inlet and a radial outlet. The influences of rotating Reynold number, cooling air Reynold number and Grashoff number on the averaged heat transfer characteristics were experimentally investigated. The experiment indicated that the averaged heat transfer coefficient on the main disk increases with the increase of cooling air Reynold number and Grashoff number, and decreases with the increase of rotating Reynold number. An experimental correlation of the averaged heat transfer coefficient on the main surface is formulated.

Key Words Swirling Discs Heat transfer Thermo-convection coefficient
Centrifugal force

NUMERICAL SIMULATION OF FILM COOLING IN ROTATING BLADE TIP

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ABSTRACT Numerical simulation of film-cooling in two types of blade tip in rotating case is completed by SIMPLE method. Turbulent eddy viscosity is determined by $k-\epsilon$ two equation model, while the wall function is considered in boundary layer. The simulation shows that centrifugal and Coriolis forces affect the flow and centrifugal force also brings about buoyancy, which is different from the usual viewpoint that the rotation has no effect on heat transfer and film cooling in blade tip. The result also indicates that the cooling effectiveness of the rotating blade is better than that of non-rotating one in the range considered.

Key Words Rotating Airfilm cooling Blades Numerical simulation

ANALYSIS OF EDGE CONTACT FOR HYPOID GEAR AND SPIRAL BEVEL GEARS

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ABSTRACT The simulation of gear meshing is performed by TCT programs in the light of the assumption that gear tooth surfaces are in contact at every instant. But the edge contact may occur due to misalignment, deflection of teeth and unsuitable machine-tool settings. So the vibration and noise of gear drives are inevitable. A new simulation method of the edge contact for hypoid gear and spiral bevel gears is provided. The contact patterns

and transmission errors for the aligned and misaligned gear drives are obtained in an example of spiral bevel gears. Although the proposed method deals with the hypoid gear and spiral bevel gears, the basic idea is suitable for spur and helical gears.

Key Words Spiral bevel gears Edge Contact Drives Errors

KINETIC CHARACTERISTICS OF TRANSMISSION TEST EQUIPMENT FOR SPIRAL BEVEL GEARS

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ABSTRACT The transfer matrix method of the lateral/torsional coupling is applied to calculation of kinetic characteristics of transmission test equipment for spiral bevel gears. The relationships of the kinetic characteristics for the test rotor are investigated in the engagement of the spiral bevel gears. The forces on the bearing and gear-box vibration are analyzed, all calculated results are consistent with the test data.

Key Words Gear drives Kinetic characteristics Bearings Radial forces Vibrations

A STUDY ON LUBRICATION OF SPIRAL BEVEL GEARS IN AEROENGINE

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ABSTRACT The vector method of the gear engagement theory is used to describe the manufacturing and meshing process of the spiral bevel gears in the aeroengine. The geometry and kinematic parameters of each contact point are considered, including the position, the curvature and the speed of each contact point on the tooth surface. The load on tooth in each contact position is determined with the help of Loaded Tooth Contact Analysis (LTCA) and the thickness of oil film on each contact position is obtained as theoretical foundation for judging the lubrication state. Meanwhile, 300 hours test of the gears has been made and the topographical feature of the tooth surface by means of scanning electron microscope is investigated. The results of calculation and experiment show that the gears are under boundary lubrication and partial elasto-hydrodynamic lubrication.

Key Words Aircraft engine Spiral bevel gears Lubrication Friction

COMPUTATION OF VISCOELASTICAL DAMPING STRUCTURE ATTACHED TO GEARS

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ABSTRACT A computational method is developed to solve engineering prob-